A STABILIZED ELECTRICAL ARC COLUMN OF VARIABLE RADIUS

A. I. Dautov, R. Kh. Ismagilov, and Kh. M. Shavaliev

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Formulas are obtained which permit calculation of the electrical and thermal characteristics of an arc column, their dependence on the physical properties and flow rate of the gas, the channel dimensions, and the initial heat conductivity function distribution.

A study of an arc column stabilized by a wall with varying channel radius is of interest in the field of optimal plasmotron design. In the initial segment of the plasmotron arc chamber the arc column, stabilized by the gas, interacts strongly with the flow, as a result of which its radius along the z axis also varies. Thus the study of a variable radius column is of practical importance. The first step in this problem, with consideration of variability in ζ was taken in [1], where a solution was obtained in the form of a series for $z \ll 1$. This present study will investigate the effect of change in ζ on the column properties over the entire channel length.

The equations used in [1, 2] to describe the column properties, allowing for variation in ζ, are written in the form

$$\frac{\partial S}{\partial z} = \frac{a^2}{r} \cdot \frac{\partial}{\partial r}\left(r\frac{\partial S}{\partial r}\right) + cE^2 S, \tag{1}$$

$$I = 2\pi R^2 E\sigma_s \int_0^{\zeta} Srdr, \tag{2}$$

where $a^2 = \pi l/G_0 h_S$, $c = \pi R^2 \sigma_S l/G_0 h_S$, $h_S = \partial h/\partial S = \text{const}$, $\sigma_S = \partial\sigma/\partial S = \text{const}$, $h - h_* = h_S S$, $\sigma = \sigma_S S$, $S = S_1 - S_*$, h_* is the value of h at $S_1 = S_*$. For solution of these equations it is necessary to know initial and boundary conditions

$$S(r, 0) = \varphi(r), \quad S_r(0, z) = 0, \quad S(\zeta, z) = 0 \tag{3}$$

and the rule for change in ζ along the channel. We will seek a solution for the particular case $\zeta^2 = 1 + kz$.

Equation (1) supposes the constancy of mass flow along the z-axis through a unit cross sectional area of the channel and the absence of radial plasma flow. However, the mass flow G through the column does vary along the z axis, and is equal to $G = G_0\zeta^2$. The change in G in the arc column stabilized by a flow of cold gas occurs because of the flow of gas moving along the z axis through the lateral surface of the column. If the column is stabilized by a wall, the variability of G occurs because of transmission of gas through the channel wall.

A solution of Eq. (1) for an arbitrary change in E along the z axis, satisfying the conditions of Eq. (3), will be given by

$$S = \exp\left(\int_0^z cE^2 dz\right) \sum_{n=1}^{\infty} A_n (1 + kz)^{-\frac{a^2\mu_n^2}{k}} \Phi_n\left(\frac{r}{\sqrt{1 + kz}}\right), \tag{4}$$

where the function

$$\Phi_n(x) = 1 + \sum_{m=1}^{\infty} a_{2m}^n x^{2m},$$

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$$a_{2m}^n = (-1)^m \frac{\left(\mu_n^2 + \frac{k}{a^2}\right)\left(\mu_n^2 + \frac{2k}{a^2}\right)\cdots\left[\mu_n^2 + (m-1)\frac{k}{a^2}\right]}{2^{2m}(m!)^2}$$

is a solution, finite at x = 0, of the equation

$$\frac{1}{x}\cdot\frac{d}{dx}\left(x\frac{d\Phi}{dx}\right) + \frac{kx}{2a^2}\cdot\frac{d\Phi}{dx} + \mu^2\Phi = 0, \tag{5}$$

μ_n are roots of the equation $\Phi(1) = 0$, A_n are the Fourier coefficients of the expansion of $\varphi(r)$ in a series in the functions $\Phi(r)$ in the interval $0 \le r \le 1$.

For brevity, we introduce the notation

$$F = \sum_{n=1}^{\infty} A_n (1+kz)^{-\frac{a^2\mu_n^2}{k}} \zeta^2\gamma_n, \quad \gamma_n = \int_0^1 \Phi_n(x)\,x\,dx.$$

From Eqs. (2, 4) we find the nonlinear integral equation for E

$$I = 2\pi R^2\sigma_s EF \exp\left(\int_0^z cE^2 dz\right),$$

whose solution will be [1]

$$E = I\left[F^2(4\pi^2R^4\sigma_s^2 + 2cI^2\psi)\right]^{-0.5}, \quad \psi = \int_0^z \frac{dz}{F^2}. \tag{6}$$

Knowing E, we find the formula for calculation of the conductivity function distribution

$$S = \frac{1}{2\pi R^2\sigma_s EF}\sum_{n=1}^{\infty} A_n(1+kz)^{-\frac{a^2\mu_n^2}{k}}\Phi_n\left(\frac{r}{\sqrt{1+kz}}\right) \tag{7}$$

the mean values of S and h

$$S_c = \frac{I}{\pi R^2\zeta^2\sigma_s E}, \quad h_c = h_* + \frac{h_s I}{\pi R^2\zeta^2\sigma_s E} \tag{8}$$

and the heat loss

$$q = 2\pi S_r(\zeta, z). \tag{9}$$

Equations (6-9) permit calculation of all the electrical and thermal properties of the column, their dependence on the physical properties and flow rate of the plasma, the channel dimensions, the initial distribution S, and the change in column radius with length.

We will consider the properties of a column using the formulas obtained. The equation of conservation of energy Eq. (1) after several transformations reduces to the form

$$\frac{h_s}{l}\cdot\frac{d}{dz}(GS_c) = q + EI. \tag{10}$$

From Eqs. (8, 10), considering that $G = G_0\zeta^2$, we find

$$\frac{dE}{dz} = -\frac{\pi R^2\sigma_s l E^2}{h_s G_0 I}(q + EI). \tag{11}$$

From Eq. (11) we find that the direction of the change in electric field intensity is determined by the sign of the quantity $\theta = q + EI$, which is the energy acquired by the plasma per unit arc column length. If $\theta > 0$, then, as in the case of a constant radius column [3], the electric field intensity in the z direction decreases. In a column of constant radius, with decrease in E the values of S_c increases [3]. In the present case, as is evident from Eq. (8), the condition for growth of S_c is decrease in the product $\zeta^2 E$. Equation (8) shows that if the condition $S_c\zeta^2$ = const is fulfilled, the electric field intensity along the z axis does not change. Thus, in contrast to the constant radius column, in the present more general case the mode E = const may occur also in the initial portion of the arc. Experiments have confirmed the existence of such a mode of arc in plasmotrons [4].

We reduce Eq. (6) to the form

$$E = \left[F^2\left(\frac{4\pi^2R^4\sigma_s^2}{I} + 2c\psi\right)\right]^{-0,5}. \tag{12}$$

Here $F^2 > 0$, $\psi > 0$. From this it is evident that if ζ, and consequently, F and ψ, are independent of I, then with increase in current the electric field intensity increases. Thus, an ascending E—I characteristic is obtained when the column radius is independent of current. This agrees with the qualitative description of the phenomenon in [5] and has been confirmed by many experiments. From Eq. (12) it also follows that decrease in E with growth in current at small I is to be explained by increase in column radius. The condition for decrease in E with growth in current, considering the dependence of ζ on I, obtained from Eq. (12), is written as

$$\frac{d}{dI}\left[\left(\frac{4\pi^2R^4\sigma_s^2}{I} + 2c\psi\right)\ln F^2\right] > 0.$$

We will consider several special cases of the solution obtained.

1. K = 0, $\zeta = 1$. Equation (5) reduces to a zeroeth order Bessel equation. Consequently, $\Phi_n = J_0(\mu_n' r)$

$$\lim_{k\to 0}(1 + kz)^{-\frac{a^2\mu_n'^2}{k}} = \exp(-a^2\mu_n'^2 z).$$

Thus, Eq. (4) can be written as

$$S = \exp\left(\int_0^z cE^2dz\right)\sum_{n=1}^{\infty} A_n \exp(-a^2\mu_n'^2 z)J_0(\mu_n' r). \tag{13}$$

Equation (13) coincides with the formula of G. Yu. Dautov's theory [3] for an arc in a cylindrical channel with a gas flow. In the still more particular case $\varphi(r) = 0$, Eq. (13) reduces [3] to the Stine—Watson solution [2].

2. G = 0. In this case $a^2 = \infty$ and Eq. (5) reduces to a zeroeth order Bessel equation. Therefore $\Phi_n = J_0(\mu_n' r/\zeta)$. In Eq. (6) for $G \to 0$ the value of F tends to zero, and $c\psi$, to infinity. We then obtain

$$E = \frac{\mu_1'}{R\zeta\sigma_s^{0,5}}, \quad S = A_1 J_0\left(\frac{\mu_1' r}{\zeta}\right). \tag{14}$$

Equation (14) agrees with the results of Mecker's theory [6] for a cylindrical flowless arc. Thus, the properties of a flowless column of variable radius ζR at every section coincide with the properties of a flowless cylindrical arc of the same radius ζR. However, it is necessary to consider that such a conclusion can be assumed to be valid only for slow changes in radius, when heat transfer in the direction of the z axis due to conductivity can be neglected.

3. $\varphi(r) = A_1\Phi_1(r)$. Then:

$$E = I\zeta^{\frac{2a^2\mu_1^2}{k} - 2}B^{-0,5},$$

$$B = 4\pi^2R^4\sigma_s^2A_1^2\gamma_1^2 + \frac{2cI^2}{2a^2\mu_1^2 - k}\left(\zeta^{\frac{4a^2\mu_1^2}{k} - 2} - 1\right), \tag{15}$$

$$S = \frac{B^{0,5}\zeta^{-\frac{2a^2\mu_1^2}{k}}}{2\pi R^2\sigma_s\gamma_1}\Phi_1\left(\frac{r}{\zeta}\right). \tag{16}$$

Equations (15), 16) in the more particular case $A_1 = 0$, k = 0 transform to the formulas of the theory expanded in [2].

Thus formulas have been obtained for calculation of properties of a variable radius column. The theories of stabilized arcs known in the literature to date are particular cases of the more general solution obtained here.

NOTATION

I	is the current, A;
E	is the electric field intensity, V/m;
l	is the arc column length, m;
R	is the initial radius of arc column, m;
r, z,	are the cylindrical coordinates related respectively to R and l (z axis directed in the direction of plasma flow);
ζ	is the radius of arc column referred to R;
σ, h, S	are the electrical conductivity, enthalpy, and thermal conductivity, 1/Ω, J/kg, W/m;
q	is the heat liberated per unit column length in radial direction per unit time, W/m;
G	is the gas mass expenditure, kg/sec;
J_0	is the zeroeth order Bessel function;
μ_n'	is the roots of the zeroeth order Bessel function.

Indices

r, z	indicate differentiation with respect to r and z;
c	is the mean mass value;
0, ∞	are the values at $z = 0$ and $l = \infty$.

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